

ANALYZING AND FORECASTING CLEAR-AIR TURBULENCE PROBABILITIES OVER THE UNITED STATES¹

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ABSTRACT

Pilot reports from special turbulence-reporting periods were used to investigate methods of analyzing and forecasting clear-air turbulence over the United States. Meteorological analyses for the special reporting periods were made objectively by computer using only standard upper air rawinsonde measurements. The wind analyses were built upward from the 400-mb level to the 200-mb level using thermal wind shears to compensate for missing wind data in high-speed portions of the flow. The best meteorological indicators of turbulence were found to be the vertical vector wind shear and the product of wind shear and horizontal deformation. To a somewhat lesser extent, large gradients of relative humidity (at the 400-mb level) and large magnitudes of divergence also tended to be associated with turbulent regions. Turbulence analyses based on both meteorological relationships and pilot reports were made. These analyses are in terms of the probability of encountering significant turbulence (moderate or severe) within a 100-n.mi. flight sector. Advection forecasts of the turbulence probabilities were made, and these showed reasonable skill for periods out to 12 hr in advance.

1. INTRODUCTION

Clear-air turbulence is an atmospheric phenomenon consisting of random three-dimensional eddies that cause aircraft to experience appreciable high-frequency accelerations. Turbulence that occurs in cloudless regions or within thin clouds such as cirrus is included in the clear-air category, while turbulence that occurs in thunderstorms and other rain-producing clouds is not. The presence or absence of turbulence in the atmosphere is not measured directly by conventional radiosonde instruments; therefore, indirect analysis methods are required. Theoretical considerations indicate that certain combinations of vertical wind shear and temperature lapse rate provide a criterion for turbulence; that is, Richardson's number. Also, it appears that atmospheric conditions must be favorable for the growth of wave motions (such as shear-gravity waves), which can eventually break into turbulent eddies (Kuettnner, 1952; Reiter and Burns, 1966). In general, turbulence tends to occur in certain portions of upper fronts and the tropopause particularly when associated with strong jet streams and sharp troughs or ridges. Also, mountain ranges are well known as being favorable locations for encountering turbulence, particularly where the lee sides are steep. Close agreement is generally found between turbulence and meteorological factors when measurements are made of the atmospheric mesostructure using research aircraft (Briggs and Roach, 1963; Endlich and

McLean, 1965; and Panofsky et al., 1968). However, practical turbulence analysis and forecasting must be based on standard rawinsonde measurements. The upper air data routinely available for the United States only permit analyses representative of regions considerably larger than the actual turbulent regions. Rather small differences in aircraft flight paths or altitudes may result in different experiences; one aircraft may encounter significant turbulence while another does not. Thus, a probability analysis of turbulence appears to be preferable to a categorical one. It is possible that pilot reports of turbulence for specified regions and time intervals could be collected in real time and combined with meteorological data in arriving at the probability analysis. In flight planning, a probability analysis will allow an individual airline or pilot to evaluate different routes and detours in terms of turbulence risk versus flight time.

2. TURBULENCE DATA

The basic turbulence data for this study were obtained from pilot reports collected over the United States during special turbulence-reporting periods. Such collections were carried out during Mar. 12-24, 1962, and Feb. 4-9, 1963; these proved to be very valuable sources of information (Colson, 1963; Colson and Panofsky, 1965). Therefore, additional turbulence-reporting periods were held during Dec. 9-14, 1964, Mar. 10-15, 1965, June 9-14, 1965, and Sept. 8-13, 1965, as part of the International Civil Aviation Organization (ICAO) worldwide high-level turbulence data collection program. The pilot

¹ This research was sponsored by the Federal Aviation Administration under Contract FA66WA-1442.

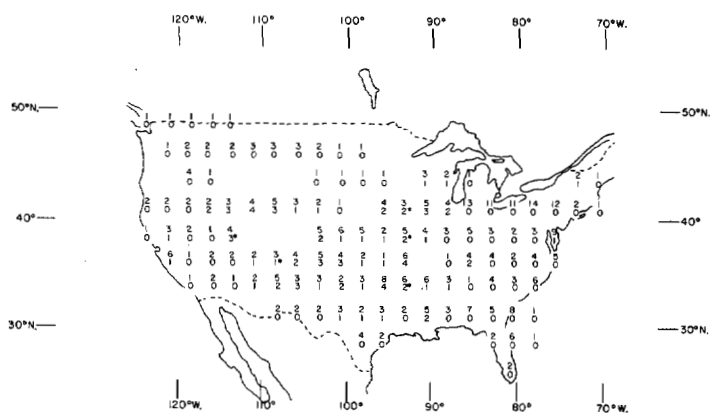


FIGURE 1.—Pilot reports collected over the United States during a 12-hr period centered on 1200 GMT, Dec. 11, 1964, 350–300-mb layer. The upper value at each point gives the total number of flight segments through a region 2.5° latitude by 2.5° longitude, while the lower value gives the number of flight segments with reports of moderate or severe turbulence; asterisks indicate occurrence of severe turbulence.

reports have been treated by subdividing the flights into segments that lie within 2.5° latitude by 2.5° longitude regions. Thus, the segments are, on the average, approximately 100 n.mi. long. A typical regional breakdown of the pilot reports for altitudes between 26,500 and 30,000 ft (350–300 mb) and during the 12-hr period of 0600–1800 GMT on Dec. 11, 1964, is shown in figure 1. The total number of flight segments compiled within a region during the 12-hr period is given by the upper value at a point, while the lower value gives the number of flight segments with reports of moderate or severe turbulence. The concentration of flying over certain areas of the United States is immediately obvious. However, the reporting of turbulence is reasonably consistent between adjacent regions.

A summary of all the turbulence data that have been used for making comparisons between turbulence and meteorological grid-point analyses is given in table 1. The table shows the total number of flight segments for each of the periods and also the percent of the totals with reports of turbulence. Turbulence was most frequently encountered during the March 1962 and the December 1964 periods (11.8 percent and 8.5 percent), although the former is based on relatively few flight segments. The lowest percentage of turbulence (2.7 percent) was during the September 1965 period. In general, the selected periods have provided a good sampling of different seasons and various synoptic conditions. The upper air synoptic patterns for the March 1962, February 1963, and December 1964 periods may generally be characterized as having well-developed wave troughs that moved eastward across the United States (Colson, 1963, 1966). A typical wind field for the December period is shown in figure 2. During the March period of 1965, a very strong upper air jet stream persisted over the United States, and wind speeds

TABLE 1.—Turbulence data used in the study (a flight segment refers to the portion of a flight that crosses a 2.5° latitude by 2.5° longitude region)

Period	Mar. 12–24, 1962 (350–300 mb)	Feb. 4–9, 1963 (400–250 mb)	Dec. 9–14, 1964 (400–200 mb)	Mar. 10–15, 1965 (400–200 mb)	June 9–14, 1965 (400–200 mb)	Sept. 8–13, 1965 (400–200 mb)
No. of flight segments	1,698	9,565	16,762	23,964	18,919	19,454
% of segments with moderate or severe turbulence.....	11.8	5.0	8.5	6.0	4.5	2.7

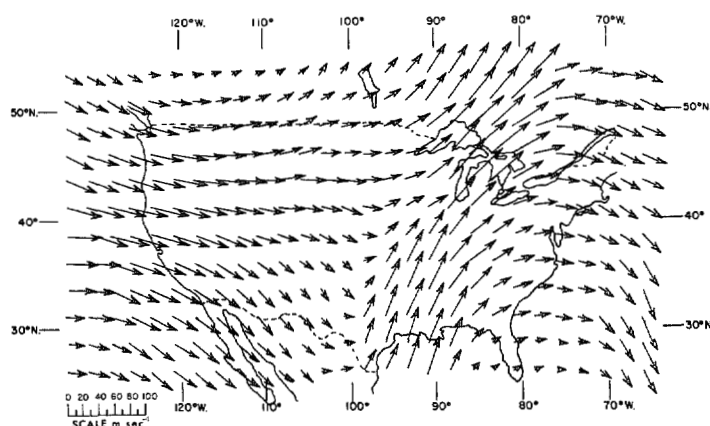


FIGURE 2.—Wind-vector analysis for 1200 GMT, Dec. 11, 1964, 350–300-mb layer.

reached 100 m sec^{-1} (fig. 8). However, during this time a blocking circulation dominated the eastern Pacific and western North America; only a few, very weak disturbances moved across the United States. Unfortunately, accurate wind analyses for both March periods and the December period were hampered by frequent missing wind data. During the June period, a weak trough tended to lie over the western United States, but there were some fairly strong upper winds (40 m sec^{-1}) for this season. During the period Sept. 8–13, 1965, the upper winds were generally light over the entire country, except in the vicinity of hurricane Betsy which entered the United States from the Gulf of Mexico.

3. OBJECTIVE ANALYSIS OF RAWINSONDE DATA

In the objective analysis technique, a grid-point value of a quantity such as temperature or a wind component is obtained by averaging the values of the five nearest observations. In the averaging process, the observed values are weighted inversely according to their distances from the grid point; that is, the nearer the observation the more important it is in determining the grid-point value. Also, the weighting factor is made larger for an observation upstream or downstream from the grid point, than for one which is an equal distance away but cross stream (Endlich

and Mancuso, 1968). This tends to align the fields with the flow direction. The above procedure has been found to give quite satisfactory results over areas such as the United States where rawinsonde data are relatively dense and evenly distributed.

When the wind-data coverage at a given altitude is complete, a wind analysis can be made independently of temperature-height data and of wind data at other altitudes. However, wind data above about 25,000 ft are often missing in the stronger jet streams. Due to this problem of missing wind observations, the basic analysis procedure was modified to build up the wind analyses from a lower level. This was done by first analyzing the grid-point wind vectors (u - and v -components) at 400 mb where the wind data are normally complete. Grid-point thermal wind-shear vectors representative of the next higher 50-mb layer are then computed based on a grid-point temperature analysis. A grid-point analysis is then made of the observed wind-shear vectors in which the thermal wind-shear vector at a grid point in question is included as a sixth observation, but with a very low weight. Thus, only in areas where the observed winds are absent does the thermal wind have a significant influence on the analysis. The 400–350-mb wind-shear analysis is added to the 400-mb wind analysis to provide a 350-mb wind field. In this manner, the wind analyses are built upward to the 200-mb level. The final wind analyses are in the form of average values for the pressure layers of 400–350, 350–300, 300–250, and 250–200 mb; the turbulence reports are subdivided similarly. An example of a grid-point wind-vector analysis for the 250–300-mb layer at 1200 GMT on Dec. 11, 1964, is shown in figure 2.

The grid-point analyses of the various observed quantities were used as the basis for computing a number of additional terms. These terms included a variety of horizontal and vertical derivatives of the wind, temperature, height, and humidity fields. Also, theoretical terms such as the Richardson number and Scorer parameter and various operationally suggested turbulence criteria were computed from the basic analyses.

4. METEOROLOGICAL INDICATORS OF CLEAR-AIR TURBULENCE

In the literature, clear-air turbulence has generally been descriptively associated with upper frontal zones, the tropopause, jet streams, and various locations within troughs or ridges of the flow. However, objective turbulence indicators are needed that can be computed from upper air data. A large number of objectively computed meteorological quantities were tested as possible turbulence indicators. Table 2 summarizes the results for the quantities that were found to be the most accurate indicators of turbulence. The table shows the percentage of turbulence that was correctly analyzed when the larger magnitudes of a meteorological quantity were used to provide a categorical "yes-or-no" analysis. The critical

TABLE 2.—Percentage of moderate or severe turbulence reports accurately analyzed using the larger magnitudes of various meteorological quantities as turbulence indicators (20 percent of all flight segments are analyzed as being turbulent for each entry). RH indicates relative humidity.

Basis of analysis	Period						
	Mar. 12–24, 1962	Feb. 4–9, 1963	Dec. 9–14, 1964	Mar. 10–15, 1965	June 9–14, 1965	Sept. 8–13, 1965	Com- bined periods
Def. $\Delta V/\Delta z$	44	52	31	34	24	34	35
$\Delta V/\Delta z$	37	39	27	40	25	34	34
Def. †	36	47	28	22	22	25	30
$\nabla \cdot \mathbf{V}$	-----	23	25	27	26	23	26
∇ RH (400 mb)	-----	-----	23	28	26	31	26

$$\dagger \text{Def.} = [(\Delta u/\Delta x - \Delta v/\Delta y)^2 + (\Delta v/\Delta x + \Delta u/\Delta y)^2]^{1/2}$$

magnitudes of a quantity that, when exceeded, would indicate turbulence were chosen so that exactly 20 percent of the total flight segments would be analyzed as being turbulent. Thus, if a turbulence indicator had "no skill" it would be expected by chance to analyze correctly 20 percent of all the turbulence. The table shows that the best results were obtained for the February 1963 period, when 52 percent of the turbulence was analyzed correctly using large magnitudes of the product of vertical wind shear and horizontal deformation as the criterion. (Deformation, def., may be described as a kinematic property of the flow that tends to transform an original circle of fluid into an elongated elliptical shape. It is an important factor in producing or destroying horizontal gradients of temperature and vertical wind shear.) The product of wind shear and deformation also provided the best results for the March 1962 and December 1964 periods. The results for those three periods were generally associated with distinct wave troughs that moved rapidly across the country. The superiority of results for February 1963 may be partially due to the fact that there were only a few missing wind observations, thus permitting optimum wind analyses. During the other three periods (March, June, and September of 1965), deformation did not add significantly when combined with wind shear. For the March 1965 period in particular, the best indicator of turbulence was wind shear alone (see also Colson, 1966). In general, the best indicators have been either wind shear or the product of wind shear and deformation. For the combined periods, they both analyzed correctly about 35 percent of all the moderate or severe reports; this is 15 percent more than would be expected by chance.

In each of the periods, turbulence tended to be more frequently associated with either large positive or large negative values of divergence; thus, the absolute value of divergence was used in preparing table 2. Another quantity that was found to be related to turbulence was the gradient of relative humidity at 400 mb. This level was used since

it was the highest at which observations of humidity were consistently available. Possibly, better results might be obtained if humidity data at the actual levels were available for making the comparisons. The relationship of vorticity to turbulence is not shown in the table and was somewhat ambiguous. For the March 1962, February 1963, December 1964, and June 1965 periods, turbulence was more frequently associated with large positive values of relative vorticity. Also, the turbulence frequencies associated with wind shear or deformation were generally found to be slightly greater when restricted to cases of positive vorticity. However, for the March 1965 and September 1965 periods, turbulence was more frequently associated with negative vorticity.

A number of other quantities were also computed and compared with the turbulence data of these periods. Among these were wind speed, lapse rate, vertical motion (kinematical), Showalter's index, tropopause height, and various terms based on geostrophic and thermal winds. However, these and other quantities tested were found to provide inferior relationships with turbulence to those shown in table 2. Also, Richardson's number, which relates turbulence to large vertical wind shears and to unstable temperature-lapse rates, was extensively tested. As shown in table 2, vertical vector wind shear relates relatively well to turbulence. However, lapse rate has not been found to add significantly when combined with wind shear. Mountain effects were also investigated by separating the data into mountainous and nonmountainous categories. During the March 1962, February 1963, and December 1964 periods, the turbulence frequencies associated with various values of meteorological quantities (wind shear and wind shear times deformation) tended to be approximately one-third greater for the mountain categories. However, during the other three periods, this separation did not show any significant mountain influence, possibly due to a low incidence of mountain waves.

5. TURBULENCE PROBABILITIES

The meteorological factors discussed in the previous section can be used to provide estimates of the turbulence probability for a given region. These estimates can then be modified according to pilot reports available at the time of analysis. If only a few flights are made through a region, the reports are not statistically reliable, and a small weight should be given to a probability value based on the pilot reports relative to the weight given to the meteorological probability. However, on the main flight routes, as many as 100 flights may traverse certain regions within a 12-hr period. For these regions with high flight densities, the analyzed turbulence probability should depend almost entirely on the pilot reports. Therefore, a turbulence probability (TP) can be determined as a weighted average of the meteorological probability (MP) and the pilots' reported probability of turbulence (RP), that is

$$TP = (W \times MP + N \times RP) / (W + N),$$

where N denotes the number of flight traverses through a region and W is an arbitrary weighting constant. When N

is greater than W , the reported probability predominates over the meteorological probability, while the reverse is true when W is greater than N . A value of 5 was chosen for W , since it was found to be the minimum value which still provided reasonable spatial consistency in the final probability distributions. The turbulence probability (TP) gives the risk of encountering significant turbulence (moderate or severe) within a 100-n.mi. flight sector. In this study, the meteorological probability (MP) was computed based on either wind shear or wind shear times deformation using relationships given in the study by Mancuso and Endlich (1966).

At present, formulas for turbulence probability such as that given above can only be tested using the special periods for which pilot reports have been collected; otherwise, one is limited to using only the meteorological probabilities. In the future, it may be possible to develop an automatic system for the continuous relaying of turbulence information from flights to a forecasting center. Also, various simplifications might be made to make systematic pilot reporting more feasible. For example, it might be possible to keep track of the number of flights through the various regions over the United States. Then only those pilots who have experienced moderate or severe turbulence would need to report in order to provide the information necessary for computing probabilities.

6. EXAMPLES OF METEOROLOGICAL AND TURBULENCE-PROBABILITY ANALYSES

In this section, charts of the various meteorological quantities and the computed turbulence probabilities are shown for a typical synoptic case (1200 GMT, Dec. 11, 1964, 350–300 mb). The reported turbulence and wind analysis for this time were previously shown in figures 1 and 2. These figures showed that a rather sharp, upper air trough existed over the center of the United States and that there were numerous reports of both moderate and severe turbulence, particularly over the Mississippi Valley and in association with the westerly flow over the Rockies. The vertical wind-shear analysis is shown in figure 3. The wind-shear magnitudes are particularly large over the Rockies, but only slightly larger than surrounding values over the Mississippi Valley. The turbulence over the latter area corresponds better with deformation, which is shown in figure 4. (The product of deformation and wind shear shows the best overall agreement with turbulence in this case.) Also, the divergence and the gradient of relative humidity (at 400 mb) are shown in figures 5 and 6. These fields also tended to show larger magnitudes in the turbulent zones. A turbulence-probability analysis, which was made by combining probabilities based on a meteorological analysis (wind shear times deformation) and pilot reports, is shown in figure 7. The turbulence probabilities vary between 0 and 55 percent.

7. TURBULENCE FORECASTING

Various approaches are possible for forecasting clear-air turbulence. For example, Kronebach (1964) made computer analyses of turbulence based on the Richardson

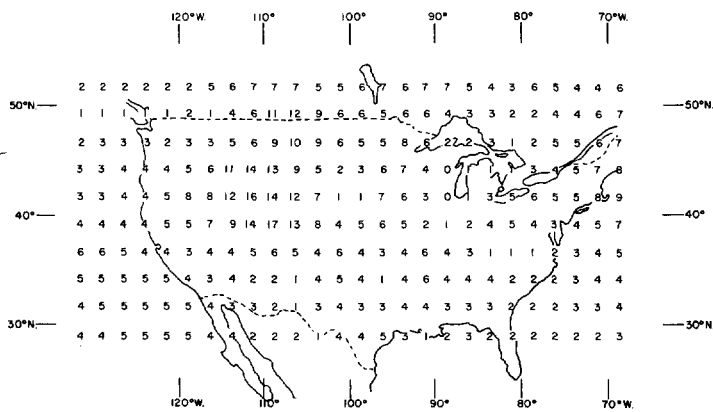


FIGURE 3.—Vertical wind-shear analysis for 1200 GMT, Dec. 11, 1964, 350-300-mb layer (units 10^{-3} sec^{-1}).

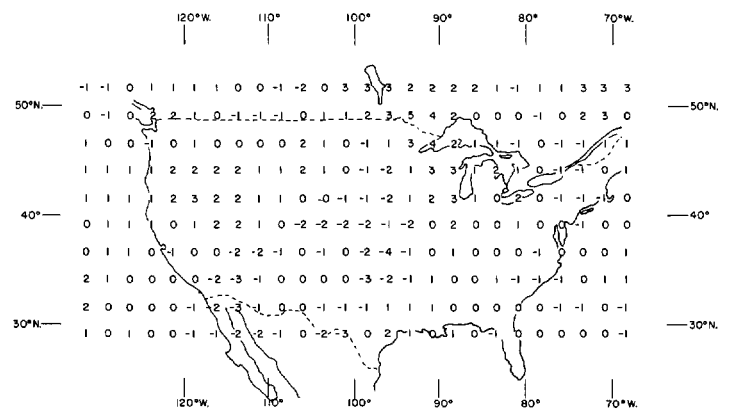


FIGURE 5.—Divergence computed from wind analysis of figure 2 (units 10^{-5} sec^{-1}).

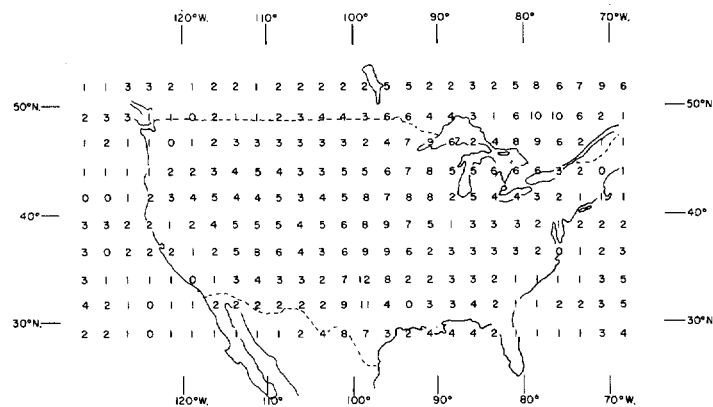


FIGURE 4.—Resultant deformation computed from wind analysis of figure 2 (units 10^{-5} sec^{-1}).

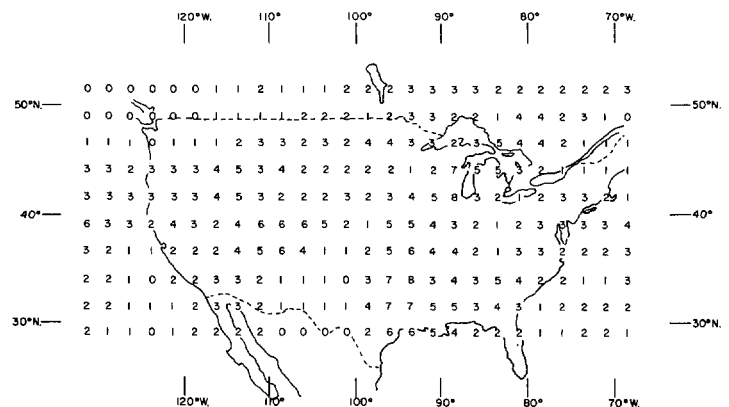


FIGURE 6.—Analysis of the relative-humidity gradient at 400 mb for 1200 GMT, Dec. 11, 1964 (units $10^{-5} \% \text{ m}^{-1}$).

number and assumed that regions analyzed as turbulent (on a categorical basis) would persist for 12 hr. Another possible method would be to first obtain a grid-point forecast of winds and then compute the quantities related to turbulence. This more complex approach appears possible in the future, but it omits the use of pilot reports. In this study, turbulence forecasts were obtained by simply transporting the turbulence probabilities with the winds in association with a barotropic type wind-forecasting scheme (Endlich and Mancuso, 1967). The turbulence advection was based on upstream difference equations (Forsythe and Wasow, 1960), which use only the center grid point in combination with the adjacent upstream grid points to perform the advection. The smoothing effects associated with upstream differencing (Molenkamp, 1968) were not found to be serious during a 12-hr forecast. Also, the standard upstream formulations were modified to include the upstream diagonal points. For example, in the case of a north-northwest wind, the differencing was made between the center grid point and the grid points both to the north and the northwest (the diagonal point). This modification was found to give more reasonable advectations at locations where the flow was largely in a diagonal direction. The advectations were computed using a 1-hr time

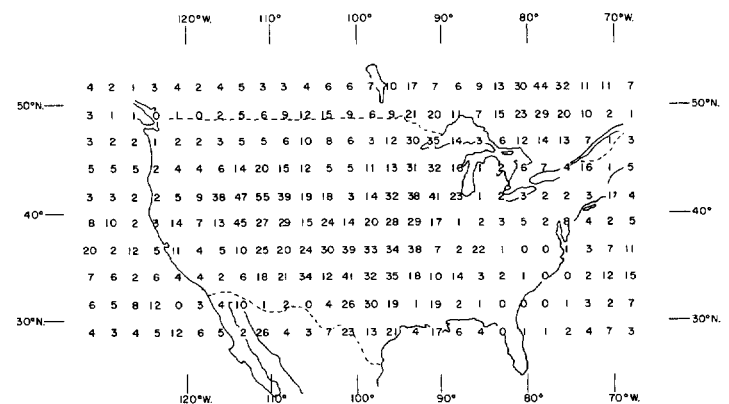


FIGURE 7.—Analysis of the probability (%) of encountering moderate or severe clear-air turbulence within a 100-n.mi. flight sector for 1200 GMT, Dec. 11, 1964, 350-300-mb layer (determined from meteorological data and pilot reports).

step and simultaneously forecast wind fields. However, the turbulence probabilities were transported only at a fraction of the upper tropospheric wind speeds. The factor used (0.5) was determined experimentally so that the turbulence probabilities would approximately maintain their

TABLE 3.—Accuracy of turbulence-probability forecasts (300–250-mb layer)

Period	Dec. 9–14, 1964	Mar. 10–15, 1965	June 9–14, 1965	Average
Correlation coefficient between forecast and analyzed changes in turbulence probabilities	0.63	0.65	0.53	0.60
Average difference between forecast and analyzed turbulence probabilities	6.5 %	6.0 %	4.8 %	5.8 %

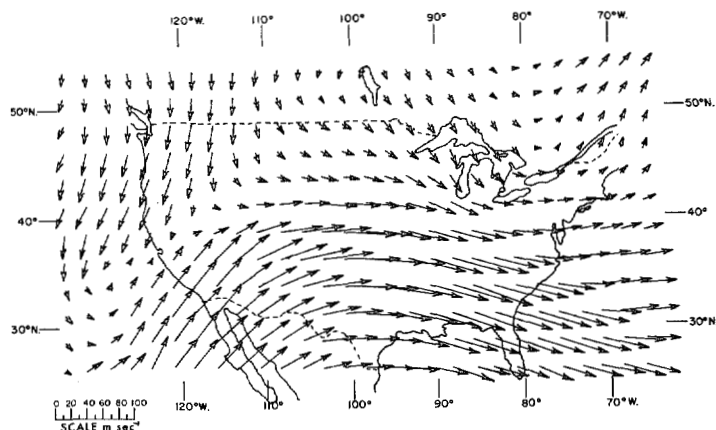


FIGURE 8.—Wind-vector analysis for 1200 GMT, Mar. 10, 1965, 300–250-mb layer.

same relative positions with respect to the synoptic features.

The above forecasting technique was found to provide reasonably accurate 12-hr displacements of the turbulence probabilities. The 12-hr turbulence forecasts were carried out using the December 1964, March 1965, and June 1965 periods as test cases. A summary of the statistics concerning the accuracy of the turbulence forecasting for the 300–250-mb layer is given in table 3. The average correlation between the forecast and actually analyzed 12-hr changes in turbulence probability was 0.6. The average difference between the forecast and analyzed turbulence percentages was about 6 percent (turbulence probabilities generally ranged between 0 and 50 percent). An example of an individual forecast is shown by figures 8 through 11. Figures 8 and 9 show the wind field and turbulence probabilities for 1200 GMT on Mar. 10, 1965 (300–250 mb). These fields were used to obtain forecast probabilities for 0000 GMT on Mar. 11, 1965 (fig. 10), which compare reasonably well with the analyzed probabilities for the same time (fig. 11). This forecast case was typical and had a correlation of 0.64 between the forecast and analyzed changes in turbulence probabilities.

8. SUMMARY

During this study an objective method of analyzing standard rawinsonde observations was developed to

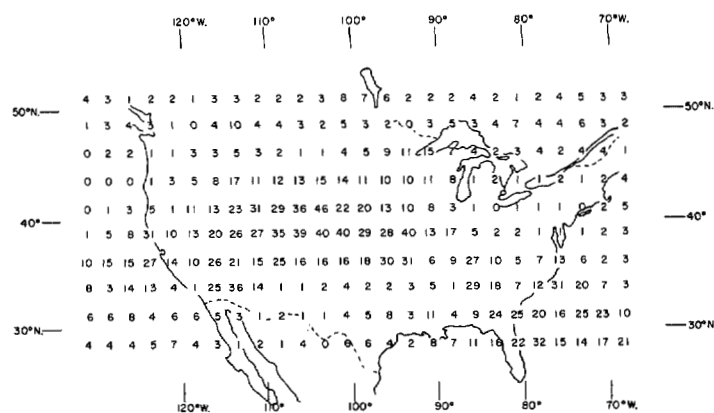


FIGURE 9.—Turbulence-probability (%) analysis for 1200 GMT, Mar. 10, 1965, 300–250-mb layer (determined from meteorological data and pilot reports).

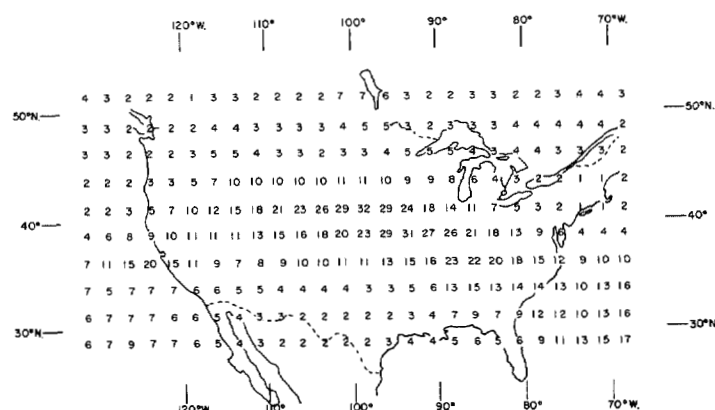


FIGURE 10.—Turbulence-probability (%) forecast for 0000 GMT, Mar. 11, 1965, 300–250-mb layer (compare with fig. 11).

obtain grid-point wind and wind-shear components. The method builds the wind analysis upward from 400 to 200 mb using both observed and thermal wind shears. The latter are given a significant weight only in areas of missing wind data. The inclusion of thermal winds proved to be important, but better relationships with turbulence were obtained when the meteorological analyses could actually be based on observed wind data. The best meteorological indicators of turbulence were found to be either wind shear (vertical) or wind shear times deformation. However, relative humidity gradients and divergence also showed consistent relationships with turbulence. The combined use of both meteorological data and pilot reports was found to be desirable for making turbulence-probability analyses. Forecasts of turbulence probabilities were made up to 12 hr in advance by advecting the probabilities with the wind field. Such forecasts were superior to persistence and gave reasonable 12-hr displacements of the turbulence probabilities. The results of the study indicate that probability analyses and forecasts can be made in order to identify low- and high-risk

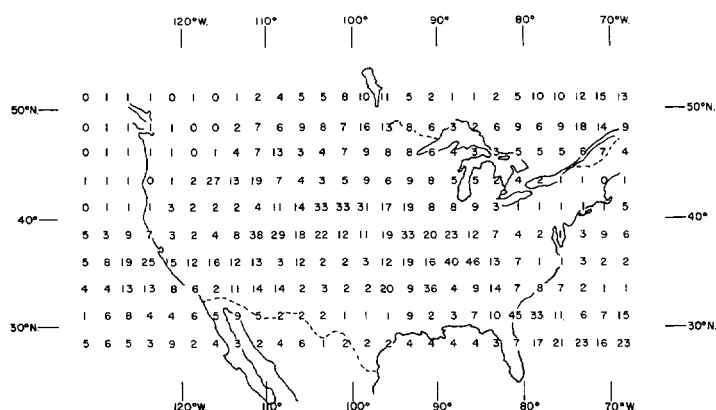


FIGURE 11.—Turbulence-probability (%) analysis for 0000 GMT, Mar. 11, 1965, 300-250-mb layer (determined from meteorological data and pilot reports).

regions of clear-air turbulence. However, accurate categorical forecasts that turbulence will or will not be encountered do not seem possible with our present observational network. A significant improvement in the analysis and forecasting of turbulence might be possible if more plentiful and accurate upper wind observations were available. Possible future sources of such wind data are wind measurements made routinely by commercial aircraft and wind measurements obtained using satellite systems for tracking constant-level balloons.

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[Received January 15, 1969; revised March 6, 1969]